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e^-e^+ Production by a Nambu String

E. B. Manoukian[1]

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We analytically calculate the probability of e^-e^+ production with a given relative energy from a closed string arising from the Nambu action as a solution of a circularly oscillating string as, perhaps, the simplest generalization of the classic pointlike particle. A numerical analysis of the result is also given.

We analytically calculate the probability (per unit period) of e^-e^+ production with a given relative energy from a closed string arising from the Nambu action (Goddard *et al.*, 1973; Kibble and Turok, 1982; Albrecht and Turok, 1989; Sakellariadou, 1990). A numerical analysis of the result is also given. The solution considered is that of a circularly oscillating closed string as, perhaps, the simplest generalization of the classic pointlike particle (Schwinger, 1973; Berestetski *et al.*, 1982; Dittrich, 1978; Pardy, 1983). The trajectory of the string is described by a vector function $\mathbf{R}(\sigma, t)$, where σ parametrizes the string. The equation of motion of the closed string is taken to be (Kibble and Turok, 1982; Albrecht and Turok, 1989; Sakellariadou, 1990; Manoukian, 1991):

$$\ddot{\mathbf{R}} - \mathbf{R}'' = 0 \tag{1}$$

with constraints

$$\dot{\mathbf{R}} \cdot \mathbf{R}' = 0, \qquad \dot{\mathbf{R}}^2 + \mathbf{R}'^2 = 1, \qquad \mathbf{R}\left(\sigma + \frac{2\pi}{\omega_0}, t\right) = \mathbf{R}(\sigma, t) \tag{2}$$

where ω_0 is some constant, and $\dot{\mathbf{R}} = \partial\mathbf{R}/\partial t$ and $\mathbf{R}' = \partial\mathbf{R}/\partial\sigma$. The general solution to (1) is

$$\mathbf{R}(\sigma, t) = \tfrac{1}{2}[\mathbf{A}(\sigma - t) + \mathbf{B}(\sigma + t)] \tag{3}$$

[1]Department of National Defence, Royal Military College of Canada, Kingston, Ontario K7K 5L0, Canada.

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where **A** and **B** satisfy, in particular, the normalization condition $\mathbf{A}'^2 = \mathbf{B}'^2 = 1$. For the system (1)-(2), we consider a solution of the form (Manoukian, 1991; Sakellariadou, 1990)

$$\mathbf{R}(\sigma, t) = \frac{1}{\omega_0}(\cos \omega_0\sigma, \sin \omega_0\sigma, 0) \sin \omega_0 t \tag{4}$$

describing a radially oscillating circular string, and for the electromagnetic current

$$\mathbf{J} = \frac{e\omega_0}{2\pi}\int_0^{2\pi/\omega_0} d\sigma\, \dot{\mathbf{R}}\delta^3(\mathbf{r} - \mathbf{R}(\sigma, t)) \tag{5}$$

$$J^0 = \frac{e\omega_0}{2\pi}\int_0^{2\pi/\omega_0} d\sigma\, \delta^3(\mathbf{r} - \mathbf{R}(\sigma, t)) \tag{6}$$

with e representing the total charge of the string. The explicit integrals in (5), (6) give ($\hbar = c = 1$)

$$\mathbf{J}(\mathbf{r}, z, t) = \frac{e}{2\pi}(\cos\theta, \sin\theta, 0)\frac{\delta(r - |\sin \omega_0 t|/\omega_0)}{r} \times \delta(z)\cos(\omega_0 t)\,\mathrm{sgn}(\sin \omega_0 t) \tag{7}$$

$$J^0(\mathbf{r}, z, t) = \frac{e}{2\pi}\frac{\delta(r - |\sin \omega_0 t|/\omega_0)}{r}\delta(z) \tag{8}$$

in cylindrical coordinates, where **r** *now* lies in the plane of the string. Using the periodicity of J^μ in time, we may write

$$J^\mu(\mathbf{r}, z, t) = \sum_{N=-\infty}^{\infty} e^{-iN\omega_0 t}\int \frac{d^2\mathbf{p}}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{dq}{2\pi} \times e^{i\mathbf{p}\cdot\mathbf{r}}e^{iqz}B^\mu(\mathbf{p}, N) \tag{9}$$

The properties of the inverse Fourier transform in (9) have been studied in detail in Manoukian (1991) and its expression is readily worked out to be

$$B^0(\mathbf{p}, N) = e(-1)^{N/2}\cos\left(\frac{N\pi}{2}\right)J^2_{N/2}\left(\frac{p}{2\omega_0}\right) \tag{10}$$

$$\mathbf{B}(\mathbf{p}, N) = \frac{e\omega_0}{p}(-1)^{N/2}N\cos\left(\frac{N\pi}{2}\right) \times (\cos\phi, \sin\phi, 0)J^2_{N/2}\left(\frac{p}{2\omega_0}\right) \tag{11}$$

where $\mathbf{p} = p(\cos\phi, \sin\phi)$, and $J_n(x)$ is the Bessel function of order n.

The probability of e^-e^+ production is given by (Schwinger, 1973; Berestetski, *et al.*, 1982; Dittrich, 1978, Pardy, 1983; Manoukian, 1986)

$$p = \int \frac{(dk)}{(2\pi)^4} J^\mu(k)^* \operatorname{Im} \bar{D}_+(k) J_\mu(k) \tag{12}$$

where $k^2 = -M^2$, $\alpha = e^2/4\pi$, and

$$\begin{aligned} \operatorname{Im} \bar{D}_+(k) &= \alpha \frac{I(M^2)}{M^2} \\ &= \frac{\alpha}{3M^2}\left(1+\frac{2m^2}{M^2}\right)\left(1-\frac{4m^2}{M^2}\right)^{1/2}, \qquad M^2 \geq 4m^2 \end{aligned} \tag{13}$$

for the lowest order contribution to the imaginary part of the photon propagator. Using the Fourier transform in (9), we obtain for the probability, per unit period (π/ω_0) of time, of e^-e^+ production with relative energy $k^0 = N\omega_0$ the expression

$$\begin{aligned} p_N = 2\pi\alpha \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k^0} \frac{dM^2}{M^2} I(M^2) \delta(k^0 - \omega_0 N) \\ \times B^\mu(\mathbf{p}, N)^* B_\mu(\mathbf{p}, N) \end{aligned} \tag{14}$$

$k^0 = (\mathbf{k}^2 + M^2)^{1/2}$. Upon writing $|\mathbf{p}| = |\mathbf{k}| \sin \Theta$ and carrying out the $|\mathbf{k}|$ integration in (14), we obtain from (10) and (11)

$$\begin{aligned} p_N = \alpha^2 \int_{4m^2}^{\omega_0^2 N^2} \frac{dM^2}{M^2} I(M^2) \\ \times \int_0^\pi d\Theta \frac{\omega_0^2 N^2 \cos^2\Theta + M^2 \sin^2\Theta}{(N^2\omega_0^2 - M^2)^{1/2} \sin\Theta} \\ \times \left(J_{N/2}\left(\frac{(N^2\omega_0^2 - M^2)^{1/2} \sin\Theta}{2\omega_0} \right) \right)^4 \end{aligned} \tag{15}$$

where the N are *even* positive integers such that $N \geq 2m/\omega_0$. Expression (15) is our main result. For $\omega_0 \to 0$, $p_N \to 0$, as expected, which would formally correspond to a point particle moving uniformly. Expression (15) is evaluated numerically for a string oscillating with angular frequency $\omega_0 = 2m$. In this case

$$\begin{aligned} p_N &= \frac{2m\alpha^2}{3} \int_1^{N^2} \frac{dx}{x}\left(1+\frac{1}{2x}\right)\left(1-\frac{1}{x}\right)^{1/2} \\ &\quad \times \int_0^\pi d\Theta \frac{N^2\cos^2\Theta + x\sin^2\Theta}{(N^2-x)^{1/2}\sin\Theta} \left(J_{N/2}\left(\frac{(N^2-x)^{1/2}\sin\Theta}{2} \right) \right)^4 \\ &\equiv \frac{2m\alpha^2}{3} \cdot 10^{-3} F(N) \end{aligned} \tag{16}$$

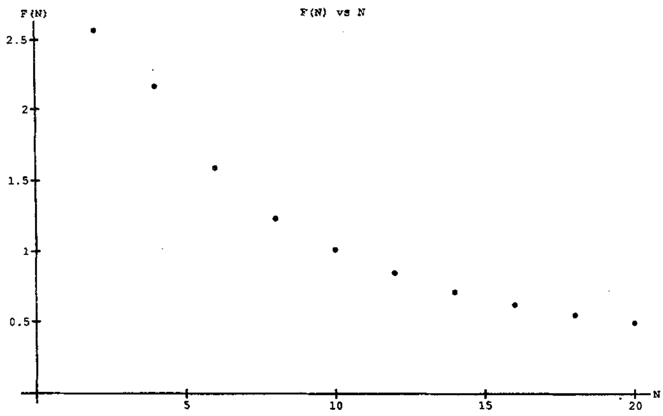

Fig. 1

$N = 2, 4, \ldots$, where $F(N)$ is plotted in Figure 1. The treatment of recoil problems of the string is beyond the scope of this paper. This approach should also provide a way of studying interacting strings via the exchange of virtual pointlike particles. These problems will be attempted elsewhere.

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